Assignment 1 – Compare three different MCDA methods

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Consult both the textbook and the lecture slides when you read this text and carry out the assignment since they complement each other. *However, the CAR method is new and does not appear in any textbook, only in the lecture slides and notes, and in the appendix.* In this assignment, you are going to analyse a decision situation with six alternatives. The alternatives should be assessed under four criteria (objectives). One of the criteria is linear, usually cost or profit (or a combination of both – net profit). The others are subjective and could be any measurable or non-numeric criteria of your choice.

Select a decision situation

Your default decision situation is to choose a Master's programme at a university. You can either keep the default decision situation and work with that or find any reasonable, real-life decision situation of interest to you. The most important aspects of your selection of decision situation are that 1) you find it interesting (since you need to spend some time thinking about it) and 2) you have access to the information and judgements necessary to model and analyse the decision.

If you keep the decision situation to choose a Master's programme at a university, you select the programmes of most interest to you from some universities of interest. If you are a last-year Bachelor's student, imagine that you will continue at the Master's level next academic year. If you are a first-year Master's student, think back one year and imagine that you are about to choose a Master's programme for the next academic year. *Do not* include the programme you are attending now since that would probably be the winner in the analysis. You should thoroughly document your selection of universities and programmes. Try not to select very similar programmes.

Document your objectives – the means objective network and your fundamental objectives hierarchy (which should contain four objectives at the lowest level, one containing a linear scale). A suggested criteria set for the Master's programme decision situation is *Cost of tuition and living*, *Quality of teaching*, *Employability*, and *Student service and facilities*. However, you are free to select any criteria you find to be appropriate. The first should be linear; the other three should be subjective.

If you do not keep the decision situation to choose a Master's programme at a university, you should thoroughly document your selected decision situation. Where did it come from, and what do the alternatives contain? Make sure there are no dependencies or overlaps between the alternatives or between the criteria. Document your objectives – the means objective network and your fundamental objectives hierarchy (which should contain four objectives at the lowest level, one containing a linear scale).

You should analyse your decision situation using each of the three methods discussed in the course: *Proportional scoring*, *Cardinal alternative ranking (CAR)*, and *Pairwise ratios*. Work with the methods one at a time, trying to elicit as accurate assessments as possible. For each of the methods, document your thinking and how you arrived at your assessments. How did you make sure your assessments were of high quality?

When you have made your assessments, enter them into the Excel model supplied with the assignment. In the first sheet (tab), enter the names of the alternatives and criteria. Each elicitation method has its own sheet where you enter your assessments.

As an example below, consider buying a car. You choose between six cars: Toyota Avensis, Volvo V70, Mercedes C280, SAAB 9-5, Ford Mondeo, and Opel Vectra. You evaluate the cars under four criteria: Price, Safety, Durability, and Image. Price has a linear scale, while the others have subjective scales.

Proportional Scoring

In the **Scoring** sheet, enter value scores for each alternative under each criterion. The value scores are numbers and should be proportional to your assessed values.

A linear scale is used for the upper left (first) criterion. You select your scale range; a suggestion is to use thousands of SEK, meaning that the linear value of SEK 150,000 would be entered as 150.

NOTE: On the linear scale, profits are positive, and costs are negative. If, for example, you have costs for tuition and living of SEK 180,000, you enter that as -180. This way, the best alternative under that criterion receives the highest (least negative) value.

For the other criteria, you use subjective scores with any scale you find appropriate. A higher score implies that an alternative is better under that criterion. If you need a starting point, you could assign a score of 100 to the best alternative and then lower scores to the rest of the alternatives. Or you could assign a score of 10 to the worst alternative and then higher scores to the rest of the alternatives.

For example, when buying a car, under the criterion Durability (which is a subjective scale) your assessment is that Toyota is the most durable, closely followed by Volvo, which in turn is clearly better than Mercedes. Thereafter comes SAAB, closely followed by Ford, and then there is a larger difference to Opel. You assign integers from 30 upwards to the alternatives proportional to your assessments of their respective durabilities. In the Excel model, it would be entered as in Table 1.

		Durability	Score
Alt.	1	Volvo V70	110
Alt.	2	SAAB 9-5	70
Alt.	3	Ford Mondeo	60
Alt.	4	Mercedes C280	90
Alt.	5	Toyota Avensis	120
Alt.	6	Opel Vectra	30

Table 1. The criterion Durability on a subjective scale assessed with proportional scores

When all the value scores have been assessed and entered for all criteria, you should next assess the weights. For each criterion in turn, consider its range; how important would it be for you to change that criterion from the worst outcome to the best (the exchange is also shown in the sheet)? Assign weight scores proportional to the importance of each criterion exchange (range).

In the car buying example, suppose that you think that the most important change would be if Price changed from worst to best, followed by Safety, which in turn is a bit more important than Image. Durability is slightly less important than Image. Then, the ranges could yield the following swing weights (but other scores consistent with the statements above are also possible):

		Swing weights	Change from (worst alt.)	Change to (best alt.)	Score
Crit.	1	Price	Mercedes C280	Opel Vectra	80
Crit.	2	Durability	Opel Vectra	Toyota Avensis	40
Crit.	3	Safety	Opel Vectra	Volvo V70	60
Crit.	4	Image	Opel Vectra	Mercedes C280	50

Table 2. Importance of the criteria ranges assessed with proportional scores

After entering the weight scores, the decision result can be found in red in the sheet. Each alternative receives a final result between 0 and 1. A higher result indicates a better choice. From the results, a ranking of the alternatives is also obtained, see Table 3.

		Results	Norm	Rank	% Opt
Alt.	1	Volvo V70	0,73	1	73%
Alt.	2	SAAB 9-5	0,66	3	66%
Alt.	3	Ford Mondeo	0,40	5	40%
Alt.	4	Mercedes C280	0,60	4	60%
Alt.	5	Toyota Avensis	0,70	2	70%
Alt.	6	Opel Vectra	0,37	6	37%

Table 3. Result of analysis with proportional scores

An optimal alternative can be formed consisting of the best outcome under each criterion (the optimal alternative, which would have a normalised result of 1, can be implicitly seen in the sheet as the column “change to” in the swing weights). The results can also be interpreted as how close (in per cent) the real alternatives are to an imaginary optimal alternative. The higher the percentage, the better the alternative.

Cardinal Alternative Ranking (CAR)

The upper left (first) criterion uses a linear scale, the same as for proportional scoring. The information is therefore copied from the Scoring sheet and need not be entered again here. For the other criteria, assess the ordering of the alternatives for each criterion in turn. For a criterion, if you could choose one alternative under this criterion, which one would that be? Put that alternative as the most preferred on top of your list (if two are equally preferred, put both on your list). Which of the remaining alternatives would you prefer? Put that as the second on your list and continue until you have a list of all alternatives in order from most to least preferred. Now, you should make this order into a cardinal order, an order that also contains the strengths. How much more or less preferred they are compared to each other? Use the following expressions:

- ’=’ equally good
- ’>’ slightly better
- ’>>’ better (clearly better)
- ’>>>’ much better

This way, you obtain the cardinal ranking of the alternatives within each criterion.

For example, when buying a car, regarding the criterion Safety, suppose that you think that a Volvo or a Mercedes (being equally safe) are clearly safer than a SAAB, which in turn is slightly safer than a Toyota. The Toyota is much safer than a Ford, which finally is slightly safer than an Opel. The cardinal alternative ranking would become *Mercedes* **=** *Volvo* **>>** *SAAB* **>** *Toyota* **>>>** *Ford* **>** *Opel*.

In the **Ranking** sheet, for each criterion except the upper left, first enter the *order* of each alternative; see the leftmost yellow column in Table 4. The order goes from best to worst, so the number representing the best alternative is entered in the topmost position. A higher position implies that an alternative is better for that criterion. Next, enter the *strength* of the ordering; see the rightmost column in Table 4. The strength indicates how strong the separation is between two ordered alternatives. The strength is expressed in the notation with ‘**>**’ symbols. These symbols are converted into steps in the following way:

- ’=’ 0 steps
- ’>’ 1 step
- ’>>’ 2 steps
- ’>>>’ 3 steps

Make this conversion for the alternatives in each criterion and enter them into the sheet. In the car example, the ranking *Mercedes* **=** *Volvo* **>>** *SAAB* **>** *Toyota* **>>>** *Ford* **>** *Opel* is converted as in Table 4. For example, the relation *Volvo* **>>** *SAAB* is entered as 2 steps in the rows between Volvo and SAAB.

Ord	**er**	**Safety**	**Steps**
Alt.	4	Mercedes C280	
			0
Alt.	1	Volvo V70	
			2
Alt.	2	SAAB 9-5	
			1
Alt.	5	Toyota Avensis	
			3
Alt.	3	Ford Mondeo	
			1
Alt.	6	Opel Vectra	

Table 4. The criterion Safety on a subjective scale assessed with cardinal alternative ranking

When you have ranked all the alternatives for all criteria, you should assess the weights next. For each criterion, consider its range; how important would it be for you to change the outcome of that criterion from the worst to the best (the exchange is also shown in the sheet). If you could only have one criterion changed from the worst (least preferred) to the best (most preferred) outcome, which one would that be? Put that criterion as the most important on top of your list (if two are equally important to change, put both on your list). Which one of the remaining criteria would you like to change? Put that as the second on your list and continue until you have a list of all criteria in importance order. Now you should make this order into a cardinal ranking, an order that also contains the strengths - how much more or less important they are compared to each other. Create a cardinal ranking between the importance of the exchanges (swings) using these expressions:

’=’ equally important
’>’ slightly more important
’>>’ more important (clearly more important)
’>>>’ much more important

When you have completed your ranking, first enter the *order* of criteria in the swing weights table in the order column; see the leftmost yellow column in Table 5. Next enter the *strengths* (importances) in the form of expressions converted to steps in the following way:

’=’ 0 steps
’>’ 1 step
’>>’ 2 steps
’>>>’ 3 steps

In the example, suppose that you think the most important change would be if Safety could be exchanged from that of an Opel to that of a Volvo. That change is clearly more important than an exchange from worst to best in Price which is the second most important exchange. An exchange in Price is much more important than an exchange in Image. The least important exchange would be in Durability, which is slightly less important than in Image. In the notation: *Safety >> Price >>> Image > Durability*. The conversion to steps is entered as the rightmost column in Table 5.

Ord	**er**	**Swing weights**	**Change from (worst)**	**Change to (best)**	**Steps**
Crit.	3	Safety	Opel Vectra	Volvo V70	
					2
Crit.	1	Price	Mercedes C280	Opel Vectra	
					3
Crit.	4	Image	Ford Mondeo	Mercedes C280	
					1
Crit.	2	Durability	Opel Vectra	Toyota Avensis	

Table 5. Importance of the criteria ranges assessed with cardinal ranking

After entering the weight ranking, the decision result can be found in the red columns in the sheet; see Figure 6. Each alternative receives a final result between 0 and 1. As with proportional scores, a higher result indicates a better choice. From the results, a ranking of the alternatives is obtained.

The result can also be interpreted as how close (in per cent) the real alternatives are to an imaginary optimal alternative. In the same manner as for proportional scores, an optimal alternative can be formed consisting of the best outcomes under each criterion (the optimal alternative, which would have the result of 1, can be implicitly seen in the sheet as the column “change to” in the swing weights).

		Results	Norm	Rank	% Opt
Alt.	1	Volvo V70	0,75	1	75%
Alt.	2	SAAB 9-5	0,66	3	66%
Alt.	3	Ford Mondeo	0,40	5	40%
Alt.	4	Mercedes C280	0,61	4	61%
Alt.	5	Toyota Avensis	0,69	2	69%
Alt.	6	Opel Vectra	0,35	6	35%

Table 6. Result of analysis with cardinal ranking

Pairwise Ratios

Even though the upper left (first) criterion uses a linear scale identical to the previous two methods, the pairwise ratio model requires the same pairwise comparisons regardless of scale type. Thus, the linear information is not copied from the Scoring sheet, and all criteria are handled similarly. For each criterion, first find the ordering of the alternatives from best to worst, in the same manner as for cardinal ranking. Next, find the strength of the ordering by considering pairwise ratios (pairwise relations) between the alternatives using the following integers to express their relative strengths: 1, 3, 5, 7, and 9, indicating that one alternative is equally good as another (strength = 1) or three, five, seven, or nine times as good. If you find this not to be expressive enough, you can also use the even integers 2, 4, 6, and 8 as intermediate values, but using only odd integers is more common.

For example, when buying a car, under the criterion Image, first establish an order. Suppose you think that a Mercedes is the best, followed by Volvo and SAAB with a gap to Toyota and Ford and Opel last. Next establish their pairwise relations. When you think about them pairwise, you find that a Mercedes is 3 times better than a Volvo or a SAAB, 7 times better than a Toyota, and so on, when it comes to the image. A Volvo is equal to a SAAB (= 1 in the table), 3 times better than a Toyota, and so on. This continues with comparing all pairs of cars until you reach the one ranked last which has already been compared to all others and obtained ratios less than 1 for all its comparisons as a consequence of being ranked last.

In the **Ratio** sheet, for each criterion, first enter the *order* of each alternative within the criterion in the same manner as for cardinal ranking; see the leftmost yellow column in Table 7. The order goes from best to worst, so the number representing the best alternative is entered in the topmost position for each criterion. A higher position implies that an alternative is better in that criterion. Next, enter the pairwise *ratios* of the alternatives. The ratio indicates the strength of the relation between two ordered alternatives. For two alternatives A and B, where A comes before B in the ordering, the ratio for the strength between A and B is entered in the row for alternative A at the column for alternative B. The integer is entered into the sheet and the inverted number (for example 1/5 if 5 was entered) is automatically inserted for the same relation going the other way (from lower order to higher).

For the car example, the following ratios were found by a decision-maker; see Table 7. For example, the strength of the relation between Alt.1 (Volvo V70) and Alt.5 (Toyota Avensis) is entered in the row of Alt.1 at the column for Alt.5 since Alt.1 is ranked higher than Alt.5. The ratio must be 1 or higher, in this case 3, and the inverse (0.33) appears automatically in the row of Alt.5 at the column of Alt.1.

Ord	er	Image	Alt. 4	Alt. 1	Alt. 2	Alt. 5	Alt. 3	Alt. 6	Score	C.I.
Alt.	4	Mercedes C280	1	3	3	7	8	9	4,07	0,89
Alt.	1	Volvo V70	0,33	1	1	3	7	9	1,99	1,17
Alt.	2	SAAB 9-5	0,33	1,00	1	3	7	9	1,99	1,17
Alt.	5	Toyota Avensis	0,14	0,33	0,33	1	3	5	0,79	1,20
Alt.	3	Ford Mondeo	0,13	0,14	0,14	0,33	1	3	0,36	1,04
Alt.	6	Opel Vectra	0,11	0,11	0,11	0,20	0,33	1	0,21	0,80
										4,6%

Table 7. The criterion Image on a subjective scale assessed with pairwise ratios

When all ratios have been entered for a criterion, it is important to consider the consistency index (C.I. in the sheet, in blue). It is a measure on how consistent the decision-maker has been in making his or her assessments. It is calculated as a function of the principal eigenvalue, but it is outside of this course to discuss the details of the calculations. If the index is below 10%, then the criterion is consistent enough to be usable. Make sure your indices stay below 10%. Otherwise, you should revise your ratio assessments.

When all ratios have been entered for all criteria, you should assess the weight ratios. In the same way as for cardinal ranking, establish an ordering of importance between the criteria. The most important criterion gets the highest position. Enter the *order* of the criteria in the order column in the weight table in the sheet. Then, unlike cardinal ranking, enter the *pairwise ratios* in the form of integers to express their relative importance: 1, 3, 5, 7, and 9, indicating that one criterion is as important as another (strength = 1) or three, five, seven, or nine times as important. If you find this not to be expressive enough, you can also use the even integers 2, 4, 6, and 8 as intermediate numbers.

After entering the weight ratios, the decision result can be found in the red sheet. Each alternative receives a final result between 0 and 1. As with the other methods, a higher result indicates a better choice. From the results, a ranking of the alternatives is obtained. The ranking order is totally comparable with the other two methods. But in the result from pairwise ratios, the resulting numbers from pairwise ratios are normalised in such a way that they sum to 1 while in the other methods, they are between 0 and 1 with a sum likely to be higher than 1. Thus, the numbers are not directly comparable with the other two methods, they will usually be lower. Also note that, unlike the two other methods, the result from pairwise ratios cannot be interpreted as how close to an optimal alternative the real alternatives are.

The report

The assignment is *individual.* Any cooperation between students is prohibited! You may only submit a report and an Excel file containing your own work and solutions.

You must document your analyses thoroughly. Your report should at least consist of:

A decision problem specification (1/2 to 1 page of text, 12 pt. Times New Roman, excluding tables and figures such as objective networks and hierarchies). Describe the context, the alternatives, and the criteria included in the problem. For the criteria, describe how they are measurable and how each scale works.

A discussion on how you solved the decision problem with each method (at least 1/2 page). How did you elicit the values and weights?

A comparison between the results obtained with the three methods (at least 1/2 page). Did the results differ (in ranking or in numerical results)? If so, in what way? Why do you think they differ? If not, how did three different methods produce the same results?

A comparison between the efforts put into working with the three methods (at least 1/2 page). Were they equally easy to work with or were there differences? If so, which was the easiest and which was the hardest? Why?

A reflection on the trade-off between effort and precision. If a method required more effort, did it give more precise results? If so, in what way?

Which method do you think would be easiest to present to others (for example a group of top managers)? Why?

Which method would you use for real-life decision problems of this size or bigger? Why?

The answers to all questions above must be well-motivated. The expected report size, excluding figures and tables, is 4-6 full pages, 12 pt. Times Roman.

Your submission must include your complete Excel file in addition to your report in Word or pdf format. Your file names must be your own name, for example Kalle_Kula.xlsx and Kalle_Kula.pdf.

Deadline: February 20, 2012

Disclosure*: While this assignment is first and foremost a part of the examination in the course, it is also one of the first empirical studies of the new CAR method that I have recently developed. Your assignment hand-ins will be collected and analysed within a research project aimed at finding new and more effective decision analysis methods. Your data will only be visible at an aggregated level in the results of the study. No information can be extracted that relates to you as an individual. Nevertheless, should you not want to be a part of the study, just state so on the first page of your assignment hand-in and I will automatically exclude you from the study. Participating in the study or not will have no effect on your grade in the course.*

Appendix: The three methods in the assignment

In this assignment, the three approaches to multi-criteria problems are discussed, and in particular how they handle imprecision in weights and alternative values. This appendix is a complement to the lectures and lecture notes in the course. We have met several ways of expressing imprecision in the course:

1. Weights (and values) can only be estimated as fixed numbers.
2. Weights (and values) can be estimated as interval statements.
3. Weights (and values) can be estimated as comparative statements.
4. Weights (and values) can be estimated as interval statements and additional comparative statements.

There are various advantages and disadvantages with all of these ways. If the expressive power of the analysis method only permits fixed number, we might get severe elicitation problems that could affect the decision quality. Imprecision is normally handled by allowing intervals, where each y_i is interpreted as an interval such that $w_i \in [y_i - a_i, y_i + b_i]$, where $0 < a_i \leq 1$ and $0 < b_i, \leq 1$ are proportional imprecision constants. Similarly, comparative statements can be represented as $w_i \geq w_j$. As seen during the lectures, then we might have an unnecessary information loss using only an ordinal ranking. When sufficiently restricting the statements involved, there is a problem with user acceptance and these methods have turned out being perceived as too difficult for most decision makers. The same trade-offs as for weights are of course valid for values as well. Expressive power in the form of intervals and comparative statements can lead to a loss of transparency on the part of the user.

Therefore, in this appendix we review the MCDM methods in the course that have a bit less expressive power but with the aim of achieving *both efficiency and user acceptance*. The question of what then constitutes a “good” method is complex, but it seems reasonably that a preferred method should possess some significant qualities to a higher degree than its rivals:

- *Efficiency*. The method should yield a valid result in as many situations as possible.
- *Easiness of use*. The steps of the method should be perceived as relatively easy to perform.
- *Ease of communication*. It should be comparatively easy to communicate the results to others.
- *Time efficiency*. The amount of time and effort required to complete the decision making task should be reasonably low.
- *Cognitive correctness*. The perceived correctness of the result and transparency of the process should be high.
- *Return rate*. The willingness to use the method again should be as high as possible.

In this appendix, our focus are on three methods that allow for a relaxation of the requirement of preciseness. We focus on the ordering of criteria weights and alternative values without make it too difficult for a decision maker to understand the process. The methods in the course are:

- Proportional scoring methods, here represented by the SMART family
- Ratio scoring methods, here represented by the widely used AHP method
- A cardinal ranking method – the new CAR method.

In the following, assume a set of criteria $\{G_1, ..., G_N\}$ where each criterion G_i correspond to a weight variable w_i. Also assume additive criteria weights, i.e. $\Sigma w_i = 1$, and $0 \le w_i$ for all $i \le N$. Further, denote the value of an alternative A_j, under criteria C_i, by v_{ij}.

Method 1: Proportional Scoring (SMART)

SMART as initially presented was a seven-step procedure for setting up and analysing a decision model. The criteria are there ranked and (for instance) 10 points are assigned to w_N, i.e., the weight of the least important criterion. Then, w_{N-1} to w_1 is given points according to the decision maker's preferences. The ranking value v_{ij} of alternative A_j is then a weighted algebraic average of the values associated with A_j:

$$E(A_j) = \sum_{i=1}^{N} w_i v_{ij} \,/ \sum_{j=1}^{N} w_{ij}.$$

In an additive model, the weights reflect the importance of one dimension relative to the others. Most commonly, the degree of importance of an attribute depends on its spread (the range of the scale of the attribute), and this is why elicitation methods like SMART, which do not consider the spread specifically, have been criticized. Yet, with methods where ranges are explicitly considered during the elicitation of weights, several empirical studies imply that people still do not adjust weight judgments properly when there are changes in the ranges of the attributes, the so called range effect. Researchers have found that decision-makers do adjust weight statements when attribute ranges vary, but that the changes are not as large as theoretically expected. Swing works like this:

- Select a scale, such as positive integers (or whatever you like)
- Consider the difference between the worst and the best outcomes (the range) within each criterion
- Imagine an alternative (the zero alternative) with all the worst outcomes from each criterion, thus having value 0 (if we have defined 0 as the lowest value)
- For each criterion in turn, consider the improvement (swing) in the zero alternative by having the worst outcome in that criterion replaced by the best one
- Assign numbers (importance) to each criterion in such a way that they correspond to the assessed improvement from having the criterion changed from the worst to the best outcome

As mentioned above an approach, which avoids some of the difficulties associated with the elicitation of exact values, is to merely provide an ordinal ranking of the criteria. It is allegedly less demanding on decision-makers and, in a sense, effort-saving. Most current methods for converting ordinal input to cardinal, i.e. convert rankings to exact surrogate weights, employ automated procedures for the conversion and result in exact numeric weights. The SMARTER (SMART Exploiting Ranks) method elicits the ordinal information on importance before being converted to numbers and thus relaxed the information input requirements from the decision maker. An initial analysis is then carried out where the weights are ordered such as $w_1 > w_2 > ... > w_N$.

To evaluate rank order, so-called ROC weights are calculated. These are weights with the properties $w_1 > w_2 > ... > w_N$, $\Sigma w_i = 1$, and $0 \le w_i$. For instance, in the case of 4 criteria and where $w_1 > w_2 > w_3 > w_4$, the weight components are $w_1 = 0.5208$, $w_2 = 0.2708$, $w_3 = 0.1458$, $w_4 = 0.0625$. The Excel sheet will do these calculations for you. Your task is to model a decision situation and enter in into the sheet.

Method 2: Cardinal Alternative Ranking (CAR)

Due to the relative robustness of linear decision models regarding weight changes, the use of approximate weights often yields reasonable decision quality, but the assumption of knowing the ranking with certainty is quite strong. Rather, there can be uncertainty regarding both the magnitudes and ordering of weights and people can be quite confident that some differences in importance are greater than others. Thus, although some weak form of cardinality may exist, cardinal importance relation information is usually not taken into account in the transformation of rank-order into weights, which may produce differences in weights that do not closely reflect what the decision-maker actually means by his/her ranking. If ordinal information as well as imprecise cardinality is taken into account, the resulting input might be more in line with a reasonable representation of significance.

For weights, the CAR method extends traditional automatic (surrogate) weights into handling cardinal information. The method also considers the order between the improvements (swing) in the criteria, but also add a concept of strength of the ordering. Thus, in the same manner as discussed during the lectures with SMARTS, the zero alternative method is used to consider the improvements. But here, the focus is on the ranking order between improvements, not numbers that should represent the improvements. If the cardinal information is omitted and only a ranking of criteria is provided, the CAR weights coincide with traditional weights (this is why the method has cardinal in its name). There is also no contradiction in the employment of CAR using swing weighting during the first step of the extraction stage (compare to the SMART method) to elicit the ordinal information on importance.

- Again, imagine an alternative (the zero alternative) with all the worst outcomes from each criterion
- For each criterion in turn, consider the improvement (swing) in the zero alternative by having the worst outcome in that criterion replaced by the best
- Rank each criterion by the assessed improvement from having the criterion changed from the worst to the best outcome
- Enter the strength of the ordering. The strength indicates how strong the separation is between two ordered alternatives. The strength is expressed in the notation with '$>_i$' symbols introduced. using the following expressions:

  ' $>_0$' equally good
  ' $>_1$' slightly better
  ' $>_2$' better (clearly better)
  ' $>_3$' much better

Furthermore, assume that there exists an ordinal ranking of *N* criteria. In order to make this order into a cardinal ranking, information should be given about how much more or less important the criteria are compared to each other. Such rankings also take care of the problem with ordinal methods of handling criteria that are found to be equally important, i.e. resisting pure ordinal ranking. In analogy with the above, we use the following expressions for the strength (cardinality) of the rankings between criteria:

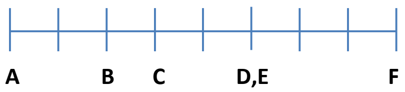
'$>_0$' equally important
'$>_1$' slightly more important
'$>_2$' more important (clearly more important)
'$>_3$' much more important

The representation idea of these two cases is now straightforward. Assume we have user induced ordering, $w_1 >_{i_1} w_2 >_{i_2} \dots >_{i_{n-1}} w_n$. Then we define a new ordering, introducing auxiliary variables x_{ij}, and define a new order containing = and > only, by substituting:

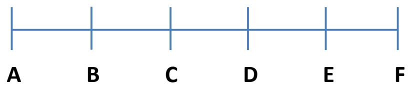
- $w_k >_0 w_{k+1}$ with $w_a = w_b$
- $w_k >_1 w_{k+1}$ with $w_a > w_b$
- $w_k >_2 w_{k+1}$ with $w_k > x_{k1} > w_{k+1}$ (*)
- ...
- $w_k >_i w_{k+1}$ with $w_k > x_{k1} > \cdots > x_{k(i-1)} > w_{k+1}$

To get some more intuition for CAR, consider the cardinality expressions (slightly more important, more important, etc.) as distance steps on an importance scale. The number of steps corresponds to the strength of the cardinalities above, by that ' $>_i$' meaning *i* steps. This can be displayed as steps on "importance rulers", where the following relationships are displayed on a cardinal (left) and ordinal (right) importance scale respectively:

- Criterion A is more important than criterion B.
- Criterion B is slightly more important than criterion C.
- Criterion C is more important than criterion D.
- Criterion D is equally important as criterion E.
- Criterion E is notably more important than criterion F.

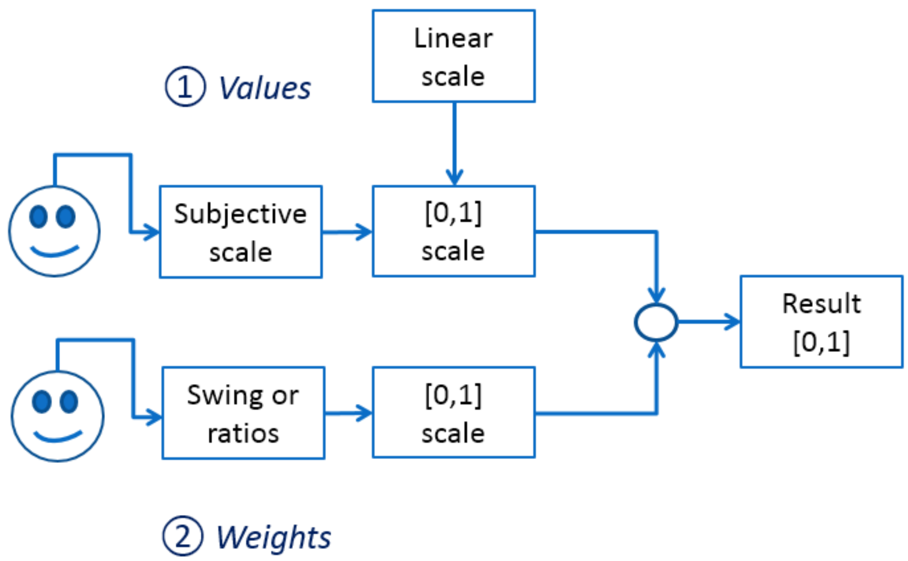


Cardinal ranking



Ordinal ranking

The decision-maker statements are converted into weights. As you can see in the Excel sheet that accompanies the assignment, the values are handled in a similar way. Finally, the values and weights are combined in the traditional way according to the multi-attribute theory of decision analysis and as explained in the lectures. The process can be illustrated as in the figure. Refer to the lecture notes and slides for more details.



Method 3: Pairwise Ratios (AHP)

The basic idea in of the Analytic Hierarchy Process (AHP) is to evaluate a set of alternatives under a criteria tree. The process requires the same pairwise comparisons regardless of scale type. For each criterion, first find the ordering of the alternatives from best to worst. Next, find the strength of the ordering by considering pairwise ratios (pairwise relations) between the alternatives using the integers 1, 3, 5, 7, and 9 to express their relative strengths, indicating that one alternative is equally good as another (strength = 1) or three, five, seven, or nine times as good. It is also allowed to use the even integers 2, 4, 6, and 8 as intermediate values, but using only odd integers is more common. Linguistically, the following interpretations are often used.

1. Equally preferred
2. Equally to moderately preferred
3. Moderately preferred
4. Moderately to strongly preferred
5. Strongly preferred
6. Strongly to very strongly preferred
7. Very strongly preferred
8. Very to extremely strongly preferred
9. Extremely strongly preferred

If criteria C_i is more important than C_j, then the reciprocal yields as well. For example, if C_1 is moderately more important than C_2, then the value 1/3 must be assigned to C_2 relative to C_1. It is then necessary to make $\frac{1}{2}n(n-1)$ pairwise comparisons. These assessments are collected in a matrix such as:

$$\begin{matrix} 1 & 0.125 & 0.333 \\ 8 & 1 & 3 \\ 3 & 0.333 & 1 \end{matrix}$$

AHP then uses matrix algebra to calculate the weights as the components in the eigenvectors associated with the maximum eigenvalues of matrices. This can be simplified by the following:

1. Multiply the values in each row together and calculate the n:th root of this product, i.e., the geometric mean
2. Normalise the results to get the weights
3. Calculate the consistency index (C.I.)

AHP also defines a consistency index (C.I.), based on the eigenvectors, indicating how consistent the decision-maker has been in making his or her assessments. It is calculated as a function of the principal eigenvalue of the matrix. If the index is below 10%, the criterion is deemed consistent enough to be usable. Otherwise, the ratio assessments must be revised. See the lecture notes for how to do that.